

PS introduction to mathematical logic

Exercises week 12

January 13, 2017

1. Prove that $\mathbb{R} \times \mathbb{R} \approx (0, 1) \times (0, 1) \preceq (0, 1) \approx \mathbb{R}$.
2. If A is well-orderable and $f : A \rightarrow B$ is onto, then B is well-orderable and $|B| \leq |A|$.
3. If κ is a cardinal and B is a non-empty set, then $B \preceq \kappa$ if and only if there is a function $f : \kappa \rightarrow B$ onto.
4. For ordinals α , α^+ is the least cardinal greater than α . Furthermore, $\alpha^+ = \alpha + 1$ when $\alpha < \omega$, while $\alpha^+ > \alpha + 1$ when $\alpha \geq \omega$.
5. If $\xi < \zeta \rightarrow \aleph_\xi < \aleph_\zeta$. κ is an infinite cardinal if and only if $\kappa = \aleph_\xi$ for some ξ .
6. Assume that α, β are ordinals with $2 \leq \min\{\alpha, \beta\}$ and $\omega \leq \max\{\alpha, \beta\}$. Then prove that $|\alpha + \beta| = |\alpha \cdot \beta| = |\alpha^\beta| = \max\{|\alpha|, |\beta|\}$.
7. Prove within the theory Z^- (ZF without replacement and foundation) that there is an uncountable well-ordered set.