

PS introduction to mathematical logic

Exercises week 10

December 9, 2016

1. Show that there are two non-isomorphic models for the language $\{0, S\}$ with exactly two elements. Show that any two models with exactly two elements that satisfy PA 1 must be isomorphic.
2. Show that the nonnegative real number \mathbb{R}^+ , with the usual interpretation of 0, successor ($Sx = x + 1$), addition and multiplication and exponentiation (where $0^0 = 1$) satisfy PA^- . Find a formula φ such that $\mathbb{R}_0^+ \not\models \text{IND}_\varphi$.
3.
 - Show that for any formula φ with one free variable, $\mathbb{N} \models \text{IND}_\varphi$.
 - Show that, for all φ , $\mathbb{N} \models \text{IND}_\varphi$.
4. Show that $\vdash_{\text{PA}} p \mid x \wedge r \cdot s = p \rightarrow r \mid x$.
5. $\mathbb{N} \models n - \text{th prime}(p, k)$ if and only if p is $= p_k$, the k -th prime.
6. Check that for any formula φ , any variable x_k and any term τ , for any natural number n :

$$\mathbb{N} \models \text{fmsub}(\ulcorner \varphi \urcorner, \ulcorner x_k \urcorner, \ulcorner \tau \urcorner, n) \leftrightarrow n = \ulcorner \varphi(x_k/\tau) \urcorner$$