PS introduction to mathematical logic

Exercises week 1 *

October 6, 2016

1. Show that the number of blocks in every sentential formula is one greater than the number of binary connectives.

2. Let $B = \{00, 01, 10, 11\}$ be the set of blocks. Also let $K_1 = \{F_1, F_2, F_3, F_4\}$ where the F_i are the operators over sequences of 0's and 1's defined as follows:

   - $F_1(x) = 0x0$
   - $F_2(x) = 0x1$
   - $F_3(x) = 1x0$
   - $F_4(x) = 1x1$

   Also, let $K_2 = \{G_1, G_2, G_3, G_4\}$ where the G_i are also the operators over sequences of 0's and 1's defined as follows:

   - $G_1(x) = 00x$
   - $G_2(x) = 01x$
   - $G_3(x) = 10x$
   - $G_4(x) = 11x$

   Show that $C(B, K_1) = C(B, K_2)$.

3. Find how many truth assignments defined on the set of blocks $\{A_1, A_2, \ldots, A_n\}$ satisfy the following set of sentential formulas $\{\neg A_1 \vee A_2, \neg A_2 \vee A_3, \ldots, \neg A_i \vee A_{i+1}, \ldots, \neg A_{n-1} \vee A_n\}$. Remember: A truth assignment S satisfy α if $\bar{S}(\alpha) = T$ and if B is a set of formulas, S satisfy A if S satisfy α, for all $\alpha \in B$.

*Almost all the exercises are taken from *The incompleteness phenomenon*, Goldstern-Judah.

4. Give the truth table for the following formulas. Which ones are tautologies?

   (a) $(\neg(p \to q) \to \neg(q \to p)) \wedge (p \vee q)$.

   (b) $(\neg(p \wedge q)) \leftrightarrow ((\neg p) \vee (\neg q)))$.

   (c) $((p \leftrightarrow (q \wedge r)) \vee ((\neg q) \leftrightarrow (\neg p)))$.

5. Let α be a sentential formula such that the only logical connective that appears in α is $\leftrightarrow$. Show that if each block that appears in α appears an even number of times, then α is a tautology. Is the converse true? Why?. (Hint: First show that $(\alpha \leftrightarrow \beta) \leftrightarrow \gamma \Rightarrow \alpha \leftrightarrow (\beta \leftrightarrow \gamma)$. Then show by induction that: if $A_{i_1}, A_{i_2}, \ldots, A_{i_k}$ are those variables which appear and odd number of times in α,then $\alpha \Rightarrow ((\ldots, (A_{i_1} \leftrightarrow A_{i_2}) \leftrightarrow \ldots) \leftrightarrow A_{i_k})$.)